Misère play: When running out of moves is a good thing

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SoIC
Background

I was teaching a games and puzzles class

I used Inquiry-based learning (IBL). students learn mathematics by tackling a carefully chosen sequence of problems provided by the instructor then students teach each other the content by presenting their solutions to each other

It's a gentle "swim or sink approach" inspired by the Moore method (R. L. Moore was a topologists who directed 50 PhD theses in the early 1900s.) This approach has been fairly successful in mathematics courses.
Combinatorial Games and Puzzles

The class was mostly about two-person games with perfect information. Many AI topics were introduced (search methods to solve puzzles, alpha-beta pruning, for example)

Students did some interesting (and not so interesting) projects. They created a Sudoku solver, AIs for card games and other games, Python implementation of games (Kalah, Texas hold ’em, Euchre)

I had suggested a project that no student wanted, so we started working on it with an UI.
**Misère play**

In a typical combinatorial game, the first player that runs out of moves loses. This is what’s called “normal” play.

In misère play, the first player that runs out of moves wins.

Our project: we are looking at misère one-symbol tic-tac-toe: The first player that completes a row, a diagonal, or a column will lose. Both player use the same symbol.

We consider square boards of arbitrary length
Super simple example

3 x 3 case is a first player wins. The first player wins if and only the middle square is covered. The rest of the perfect strategy is:

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1 2 3
4 X 1
2 3 4
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Misère play is hard

Normal play is hard (games are fun, aren’t they?)

Misère play is harder. For impartial games, the "fundamental structure" of games in normal play forms a group vs. a semigroup for misère play.

For normal play, every impartial game is "equivalent" to a NIM game (two players take turn removing coins from a pile). This is known since the late 1930s.

For misère play, an "equivalent" theory was developed in the mid 2000s.
Results thus far

Conjecture: The second player wins misère one-symbol tic-tac-toe on an n x n board, n > 2, if and only if n is even.

True for n = 4k (there is a simple “mirror” strategy)

Currently checking “bigger” cases n = 5, 6, 7. The game trees are huge…

Interesting patterns have arisen: Unlike the 5x5 board, the first player has MANY, MANY winning moves

Mirror strategy is IMPOSSIBLE to apply, so the problem is much harder if n ≠ 4k. Still working on this…
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THANK YOU

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